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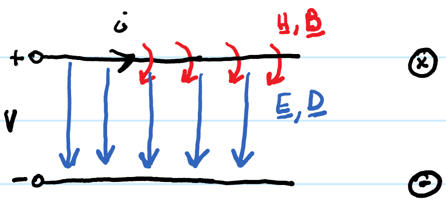

* $\underline{E}$ points from high to low voltage

⊗ = into the page

⊙ = out of the page

<u>i produces:</u>

$\underline{H}$: magnetic field. can be related to

$\underline{B}$: magnetic flux density.

<u>V produces:</u>

$\underline{E}$: electric field. can be related to

$\underline{D}$: electric displacement field.

The are related by Maxwell's equations.

<u>Integral</u>	<u>Differential</u>	
$\oint_{\partial\Omega} \underline{D}\cdot\hat{n}\, dA = \int_{\Omega} \rho_f \, dV$	$\nabla\cdot\underline{D} = \rho$	Gauss's law
$\oint_{\partial\Omega} \underline{B}\cdot\hat{n}\, dA = 0$	$\nabla\cdot\underline{B} = 0$	Gauss's law for magnetism
$\oint_{C} \underline{E}\cdot d\underline{l} = -\frac{d}{dt}\int_{\partial\Omega} \underline{B}\cdot\hat{n}\, dA$	$\nabla\times\underline{E} = -\frac{\partial \underline{B}}{\partial t}$	Maxwell-Faraday equation
$\oint_{C} \underline{H}\cdot d\underline{l} = \int_{\partial\Omega} \underline{J}_f\cdot\hat{n}\, dA + \frac{d}{dt}\int_{\Omega} \underline{D}\cdot\hat{n}\, dA$	$\nabla\times\underline{H} = \underline{J} + \frac{\partial \underline{D}}{\partial t}$	Ampere's Circuital law

Assume: $\underline{B} = \mu\underline{H}$, $\underline{D} = \varepsilon\underline{E}$

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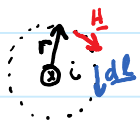
Devices of interest: inductors, transformers

* Static devices
* Need only Ampere's and Gauss's laws

  Use Faraday's law to find Voltage.

Ex|



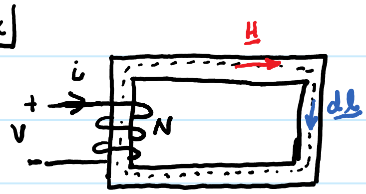
$$\oint_c \underline{H}\cdot \underline{dl} = \oint_c H\,dl = Hl$$

$$\int_{\partial\Omega} \underline{J}\cdot\hat{n}\,dA = i$$

$$Hl = i \quad \Rightarrow \quad H = \frac{i}{l} \qquad l = 2\pi r \qquad \Rightarrow \quad \boxed{H = \frac{i}{2\pi r}}$$

Field strength is related to i and decays like $\frac{1}{r}$ as expected.

Ex|



iron with permeability μ, area A

* Assume $\underline{H}$ is constant and mainly in the iron

$$\oint_c \underline{H}\cdot\underline{dl} = \oint_c H\,dl = Hl$$

$$\int_{\partial\Omega} \underline{J}\cdot\hat{n}\,dA = Ni \qquad \text{(total current enclosed)}$$

(Magneto Motive Force: MMF)

$$Hl = Ni$$

Total magnetic flux: $\phi = BA \quad \Rightarrow \quad H = \frac{\phi}{\mu A}$

$B = \mu H$

$$\phi\left(\frac{l}{\mu A}\right) = Ni$$

$$\boxed{\phi = \frac{Ni}{\frac{l}{\mu A}}}$$

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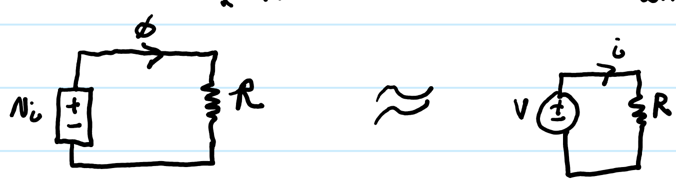
$\frac{l}{\mu A}$ resembles the equation for Resistance in an electric circuit.

$\mathcal{R} = \frac{l}{\mu A}$ is the reluctance

$\mathcal{P} = \frac{1}{\mathcal{R}}$ is the permeance.

$\phi = \frac{Ni}{\mathcal{R}} \Rightarrow \boxed{Ni = \mathcal{R}\phi}$

* Resembles a circuit with a resistor



* We call this representation an equivalent magnetic circuit

Equivalences to electric circuits:
MMF $\Rightarrow$ EMF (voltage)
$\mathcal{R} \Rightarrow R$
$\mathcal{P} = \frac{1}{\mathcal{R}} \Rightarrow \sigma$
$\phi \Rightarrow i$

* What about the $\underline{E}$ field?

$$\oint_c \underline{E} \cdot d\underline{l} = -\frac{d}{dt}\int_{\partial\Omega} \underline{B} \cdot \hat{n}\, dA$$

* Assume $\underline{B}$ is constant in space and $\partial\Omega$ is constant in time

$$\oint_c \underline{E} \cdot d\underline{l} = -\frac{d}{dt}(NBA) \Rightarrow \oint_c \underline{E} \cdot d\underline{l} = -\frac{d}{dt}(N\phi) = -V + \oint_{wire} \underline{E} \cdot d\underline{l} = -\frac{d}{dt}(N\phi)$$

* Assume σ (electrical conductivity) of the wire $\sigma \to \infty$ $(R \to 0)$.

$J = \sigma E \Rightarrow E = \frac{J}{\sigma}$. J is finite, so $E_{wire} \to 0$

$-V + 0 = -\frac{d}{dt}(N\phi) \Rightarrow V = \frac{d}{dt}(N\phi)$.

$\lambda = N\phi$ (flux linkage, flux linked to the coil). $\Rightarrow \boxed{V = \frac{d\lambda}{dt}}$

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$\lambda = N\phi \Rightarrow \lambda = N\left(\frac{Ni}{\frac{\ell}{\mu A}}\right) \Rightarrow \lambda = \left(\frac{N^2\mu A}{\ell}\right) i$

$$L \equiv \frac{N^2\mu A}{\ell} \quad \text{(the inductance)}$$

$$\lambda = Li$$

$V = \frac{d\lambda}{dt} \Rightarrow V = \frac{d}{dt}(Li) \Rightarrow V = L\frac{di}{dt} + i\cancel{\frac{dL}{dt}}^{\,0}$

$$\boxed{V = L\frac{di}{dt}}$$

Ex| $v = \sqrt{2}(120)\cos(\omega t)$ V $\quad f = 60$ Hz, $\omega = 2\pi f$

$N = 100$ turns $\quad A = 0.0049\,m^2 \quad \ell = 40$ cm $\quad \mu = 1000\mu_0 = 4\pi\times10^{-4}$ H/m

Find: i after transients have died away

Solution: $L = \frac{N^2\mu A}{\ell} \Rightarrow L = 0.154$ H

$V = L\frac{di}{dt} \Rightarrow \frac{di}{dt} = \frac{v}{L} \Rightarrow \frac{di}{dt} = \frac{\sqrt{2}(120)}{0.154}\cos(\omega t)$

$$i = \frac{\sqrt{2}(120)}{0.154\,\omega}\sin(\omega t)$$

$$\boxed{i = 2.92\sin(\omega t)\ A}$$